Zélus: a synchronous language with ODEs

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Hybrid Systems Modelers

Program complex discrete systems and their physical environments in a single language

Many tools exist

- Simulink/Stateflow, LabVIEW, Modelica, Ptolemy, ...

Focus on programming language issues to improve safety

Our proposal

- Build a hybrid modeler on top of a synchronous language
- Recycle existing techniques and tools
- Clarify underlying principles and guide language design/semantics
Typical system

Discrete controller

- Dataflow equations
- Hierarchical automata

Physical environment

- ODEs with reset
  \[ \text{der } v = (0.7 / \max) \times \text{error init } 0.0 \text{ reset hit}(v_0) \rightarrow v_0 \]

- Hierarchical hybrid automata

```plaintext
rate = 0.0

push() on (not segout)
atlimit()

rate = \max\n
atlimit = \text{up}(\text{angle} - \max)
pull()
atlimit() on (last v > 0.3 \times \max)
emit hit = -0.8 \times \text{last } v

rate = -\max\n
atlimit = \text{up}(\min - \text{angle})
pull()
atlimit()
push()

atlimit() on (last v < -0.3 \times \max)
emit hit = -0.8 \times \text{last } v
```
Reuse existing tools and techniques

Synchronous languages (SCADE/Lustre)

- Widely used for critical systems design and implementation
  - mathematically sound semantics
  - certified compilation (DO178C)
- Expressive language for both discrete controllers and mode changes

Off-the-shelf ODEs numeric solvers

- Sundials CVODE (LLNL) among others, treated as black boxes
- Exploit existing techniques and (variable step) solvers

A conservative extension:
Any synchronous program must be compiled, optimized, and executed as per usual
Type systems to separate continuous from discrete

What is a discrete step?

- Reject unreasonable parallel compositions
- Ensure by static typing that discrete changes occur on zero-crossings
- Statically detect causality loops, initialization issues

Simulation engine

\[
\sigma' = d_\sigma(t, y) \quad \text{upz} = g_\sigma(t, y) \quad \dot{y} = f_\sigma(t, y)
\]
Compiler architecture

Built on an existing synchronous compiler

- Source-to-source and traceable transformations
- Resulting program is synchronous and translated to sequential code
Comparison with existing tools

Simulink/Stateflow (Mathworks)

- Integrated treatment of automata vs two distinct languages
- More rigid separation of discrete and continuous behaviors

Modelica

- Do not handle DAEs
- Our proposal for automata will be integrated into new version 3.4

Ptolemy (E.A. Lee et al., Berkeley)

- A unique computational model: synchronous
- Everything is compiled to sequential code (not interpreted)
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Programming embedded systems and their environments in the same language

- A Lustre-like language with ODEs.
- Dedicated type systems to separate discrete time from continuous time behaviors.
- A compiler architecture based on checkable source-to-source transformations.
- Simulate with an off-the-shelf numeric solver.

Example system with (hierarchical) Hybrid Automaton

Hybrid simulation run-time

The Type system

 inconvenience

let hybrid system (rate maxf pull push bang)

bucket push bucket pull

bucket push bucket pull

zero-crossings

variable composition

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